LATTICE STUDY OF THE SIMPLIFIED MODEL OF M-THEORY

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Lattice discretization of the supersymmetric Yang-Mills quantum mechanics is reviewed and results of the Monte Carlo simulations of the simplified model are presented. The D=4, N=2, quenched system, studied at finite temperature, reveals existence of the two distinct regions which may correspond to a black hole and the elementary 0-branes phases of the M-theory conjectured in the literature. New results for higher gauge groups $N < 9$ lead to the similar picture, however the nature of the transition between the two phases and its precise scaling with N is yet unresolved.

1 Matrix quantum mechanics and its lattice formulation

Recent developments in nonperturbative string theories have lead to an exciting hypothesis of the existence of a single theory (M-theory) which encompasses five known superstring theories and eleven dimensional supergravity in a unified scheme (for a recent review see e.g. [1]). Even though its details are not known, the M-theory has a tantalizing potential to unify all interactions and particles. In particular it may offer a topological explanation of such fundamental features as three families and fractional charges. It may lead to a standard model gauge group with $\mathcal{N}$=1 supersymmetry. It provides understanding of the Bekenstein-Hawking entropy puzzle and much more. Moreover, Banks, Fishler, Susskind and Shenker [2] suggested that the spectrum of M-theory is equivalent to that of a supersymmetric Yang-Mills quantum mechanics (SYMQM) which results from the dimensional reduction of the 10 dimensional supersymmetric Yang-Mills theory. This allows to use a host of nonperturbative methods to quantitatively study both systems. Accordingly we have constructed the Wilson discretization of the above quantum mechanics and proposed to investigate it with the standard lattice techniques [3]. The ultimate goal is to study D=10 SYMQM for the large size of the SU(N) matrices. However, even for this relatively simple, one dimensional quantum mechanical system this is a formidable task, the main difficulty being the complex fermionic determinant (and pfaffian) in D=10, and time consuming procedures for simulating nonabelian systems of large matrices. On the other hand the system can be simulated at present in other regions of the parameter space (D, N, N_f) and such a study may provide some information about its general structure. Therefore we have decided to set up a systematic lattice study of SYMQM beginning with the simplest case of $D = 4, N = 2, N_f = 0$ and gradually extending it as far as possible towards the BFSS limit. In fact in this talk I will also report on some results for higher N. Supersymmetric Yang-Mills Quantum Mechanics has been intensively studied [4]. Although its exact solution is still not available, many results are known and a) can be tested and b) provide us a guidance from a "simple corner" of parameter space to the ultimate "BFSS corner".

The action of the SYMQM reads

$$S = \int dt \left(\frac{1}{2} \mathrm{Tr} F_{\mu\nu}(t)^2 + \bar{\Psi}^a(t) \mathcal{D} \Psi^a(t) \right). \tag{1}$$

where $\mu, \nu = 1 \dots D$, all fields are independent of the space coordinates $\vec{x}$ and supersymmetric fermionic partners belong to the addjoint representation of SU(N). The discretized system is put on a D dimensional hypercubic lattice $N_1 \times \ldots \times N_D$ reduced in all space directions to $N_i = 1$, $i = 1 \dots D-1$. Gauge and fermionic variables are assigned to links and sites of the new elongated lattice in the standard manner. The gauge part of the action reads

$$S_G = -\beta \sum_{m=1}^{N_t} \sum_{\mu>\nu} \frac{1}{N} Re(\mathrm{Tr}\, U_{\mu\nu}(m)), \tag{2}$$

with $\beta = 2N/a^3 g^2$, and $U_{\mu\nu}(m) = U_\nu^\dagger(m) U_\mu^\dagger(m+\nu) U_\nu(m+\mu) U_\mu(m)$, $U_\mu(m) = \exp{(iagA_\mu(am))}$, where a denotes the lattice constant and g is the gauge coupling in one dimension. The integer time coordinate along the lattice is m. Periodic boundary conditions $U_\mu(m + \nu) = U_\mu(m)$, $\nu = 1 \dots D-1$, guarantee that Wilson plaquettes $U_{\mu\nu}$ tend, in the classical continuum limit, to the appropriate components $F_{\mu\nu}$ with space derivatives absent. In this formulation the projection on gauge invariant states is naturally implemented. Eq. (2) is the basis of the MC simulations of the still more simplified, D=4, N=2 and $N_f = 0$ (quenched), model [3]. Some preliminary results for higher gauge groups $N < 9$ are now also available and will be shortly discussed.

2 Results

As a first problem we have chosen the question of the phase structure of the model. A very interesting feature of the M-theory is the solution of the Bekenstein-Hawking entropy puzzle in terms of the elementary branes [5]. In particular, the theory predicts existence of the phase transition between a low temperature "black hole" phase and a high temperature phase described by the elementary branes [6] [a]. Threfore a natural question arises if the simplified model (i.e. SYMQM) posesses any nontrivial phase structure [7]. At the same time it is well established that QCD (or pure Yang-Mills theory) has two different phases (e.g. confinement and deconfinement). Since the action (2) is basically QCD-like, one may expect that the dimensionally reduced model may indeed exhibit similar phenomenon. To check this we have measured the distribution of the trace of the Polyakov line

$$P = \frac{1}{N} \mathrm{Tr} \left(\prod_{m=1}^{N_t} U_D(m) \right). \tag{3}$$

which is a very sensitive determinant of the two phases in QCD. Similarly to lattice pure gauge system, symmetric concentration of the trace around zero indicates the low temperature phase with $< P >= 0$ (here a "black hole" phase) while clustering around ± 1 (or, for arbitrary N,

[a] The full phase structure of the M-theory is expected to be much more complex *op. cit.*

around the elements of Z_N) is characteristic of the high temperature phase (here the elementary 0-brane phase).

There is one important difference between the dimensionally reduced model and the pure gauge system in the large spatial volume. Of course the one dimensional system with local interactions cannot have a phase transition for finite N. However at infinite N the sharp singularity may occur. This suggests that the number of colours plays a role of a volume (indeed the number of degrees of freedom is proportional to N^2). Consequently, and similarly to a small volume systems, we expect to see some signatures of a phase change for finite and even small N . However the generic singularities would develop only at infinite N. In fact studying the N dependence of the above signatures allows often to determine the properties of the infinite N phase transition via the final size scaling analysis.

In Ref.[3] above program was carried out for D=4, N=2 and in the quenched approximation $N_f = 0$. It was found that the distribution of (3) is indeed changing from a convex to a concave one at some finite value of $\beta = \beta_c(N_t)$. Moreover the dependence of β_c on N_t turned out to be consistent with the canonical scaling. The best fit to Monte Carlo results gave $\beta_c(N_t) = (0.17 \pm 0.05)N_t^{(3.02\pm0.33)}$ in good agreement with the expected dependence $\beta_c \sim N_t^3$. This indicated that in the continuum limit the transition occurs at *finite* temperature $T_c = A(g^2N)^{(1/3)}$, with $A = .28 \pm 0.03$. We have also measured the average size of the system $R^2 = g^2 \sum_a (A_i^a)^2$ which is shown in Fig.1 for different values of the temperature. Our results agree qualitatively with the mean field calculation at large N [7]. Moreover, the pseudocritical temperature determined from Fig.1 is consistent with the value of A quoted above, which to our knowledge was determined for the first time. Obviously there is a long way between $N = 2$ and the BFSS limit $N \to \infty$. Even though there is an intriguing evidence from lattice simulations that large N limit may be reached rather early in N (at least for some observables) [8,9], it is very important to repeat this calculation for higher N. In particular the coefficient A may depend on N.

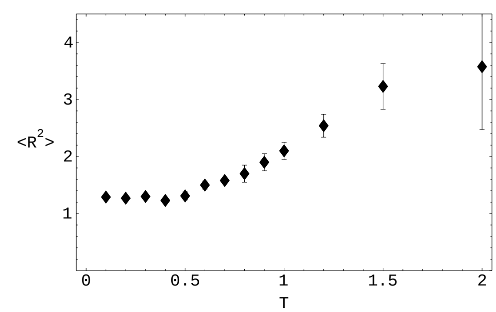

Figure 1. Extrapolated to the continuum limit size of the system as a function of the temperature (in the units of the one dimensional gauge coupling g).

Currently we are extending this analysis to higher gauge groups (and still for $N_f = 0$). Detailed account of our results will appear elsewhere. Main points are the following. For $N > 2$ the trace P in Eq.(3) is complex. The condition det(U)=1 constraints the trace to lie inside the "N-star" in the complex plane (see Fig.2). In the low temperature phase the distribution is symmetric and peaked around $P = 0$ ensuring $< P >= 0$, while in the high temperature phase one expects P's to be concentrated around the elements of Z_N. A sample of our preliminary results for N=3,5,8 and at high and low temperatures ($\beta \sim T^3$) is shown in Fig.2. Indeed one clearly sees existence of both (high and low temperature) regimes. Moreover, in the high temperature region, the system has a tendency to be stuck in one of the Z_N minima. This is a typical indication of the spontaneous symmetry breaking which might occur at $N \to \infty$ leading eventually to a non-zero value of the order parameter $< P >$. We expect to accumulate soon enough data to be able to decide if the two above regions are separated by the genuine phase transition or a smooth cross-over.

To conclude, the lattice simulations provide a novel approach to the Yang-Mills quantum mechanics and possibly to the M-theory. Many problems (e.g. identification of the graviton

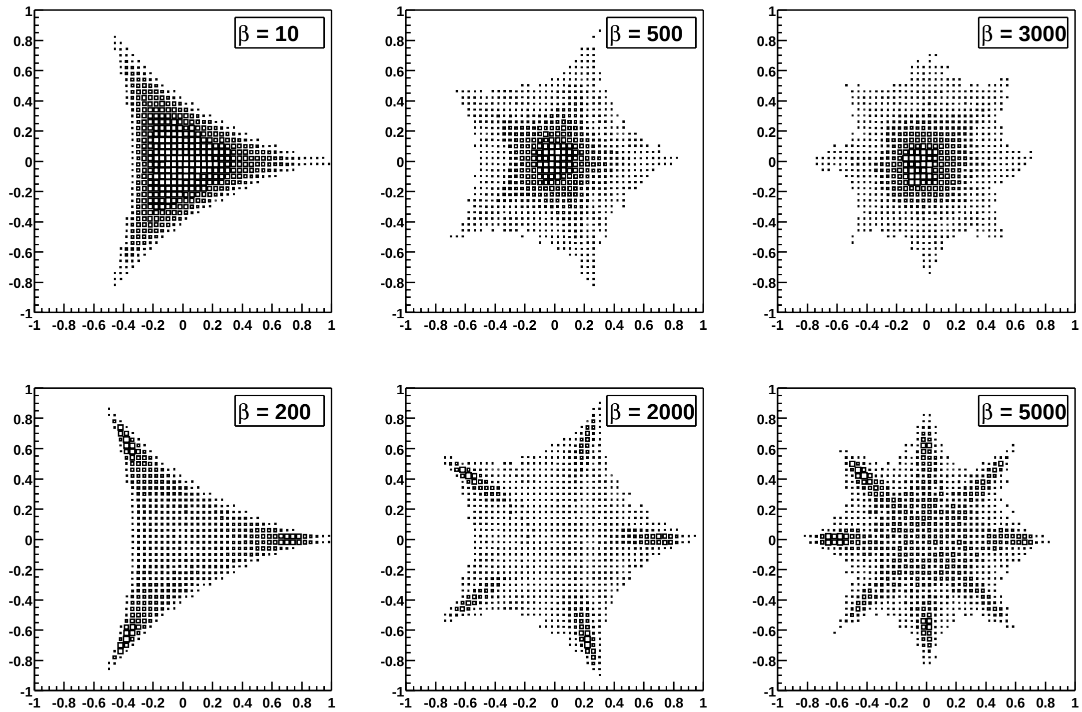

Figure 1: Distribution of the Polyakov line in the low (upper row) and high (lower row) temperature phases of the quenched Yang-Mills quantum mechanics with $N = 3, 5, 8$.

multiplet) can be formulated and quantitatively attacked in this framework. On the other hand much more work is required to continue development of this fascinating subject. In particular, construction of the new algorithms capable to deal effectively with dynamical fermions propagating in this one dimensional system is an open question.

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